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Research on Matrix Fractional Integrals

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Abstract: **In this paper, based on Jumarie type of Riemann-Liouville (R-L) fractional integral and a new multiplication of fractional analytic functions, we obtain the exact solutions of two matrix fractional integrals. In fact, our results are generalizations of classical calculus results.**

Keywords: **Jumarie type of R-L fractional integral, new multiplication, fractional analytic functions, matrix fractional integral.**

I. INTRODUCTION

Fractional calculus with derivatives and integrals of any real or complex order has its origin in the work of Euler, and even earlier in the work of Leibniz. Shortly after being introduced, the new theory turned out to be very attractive to many famous mathematicians and scientists, for example, Laplace, Riemann, Liouville, Abel, and Fourier. Fractional calculus has important applications in many scientific fields such as physics, mechanics, biology, electrical engineering, viscoelasticity, control theory, economics, and so on [1-13].

However, the rule of fractional derivative is not unique, many scholars have given the definitions of fractional derivatives. The common definition is Riemann-Liouville (R-L) fractional derivative. Other useful definitions include Caputo fractional derivative, Grunwald-Letnikov (G-L) fractional derivative, and Jumarie type of R-L fractional derivative to avoid non-zero fractional derivative of constant function [14-18].

In this paper, based on Jumarie type of R-L fractional calculus and a new multiplication of fractional analytic functions, we solve the following two matrix fractional integrals:

$$\left({}_{-\infty}I_{x}^{\alpha}\right)\left[E_{\alpha}(rAx^{\alpha})\otimes_{\alpha}\left(cos_{\alpha}(sAx^{\alpha})\right)^{\otimes_{\alpha}p}\right],$$

and

$$\left({}_{-\infty}I_{x}^{\alpha}\right)\left[E_{\alpha}(rAx^{\alpha})\otimes_{\alpha}\left(sin_{\alpha}(sAx^{\alpha})\right)^{\otimes_{\alpha}p}\right],$$

where $0<\alpha\leq 1, (-1)^{\alpha}$ exists, r, s are real numbers, $r>0$, p is a positive integer, A is a real matrix, and A is invertible. In fact, our results are generalizations of traditional calculus results.

II. PRELIMINARIES

Firstly, we introduce the fractional calculus used in this paper.

Definition 2.1 ([19])**:** Let $0<\alpha\leq 1$, and x_0 be a real number. The Jumarie's modified Riemann-Liouville (R-L) α-fractional derivative is defined by

$$\left({}_{x_0}D_{x}^{\alpha}\right)[f(x)]=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{x_0}^{x}\frac{f(t)-f(x_0)}{(x-t)^{\alpha}}dt\,, \quad (1)$$

And the Jumarie type of Riemann-Liouville α-fractional integral is defined by

$$\left({}_{x_0}I_{x}^{\alpha}\right)[f(x)]=\frac{1}{\Gamma(\alpha)}\int_{x_0}^{x}\frac{f(t)}{(x-t)^{1-\alpha}}dt\,, \quad (2)$$

where $\Gamma(\ )$ is the gamma function.

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Proposition 2.2 ([20])**:** *If* α,β,x_0,C *are real numbers and* $\beta\geq\alpha>0$*, then*

$$\left({}_{x_0}D_x^{\alpha}\right)\left[(x-x_0)^{\beta}\right]=\frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(x-x_0)^{\beta-\alpha}, \tag{3}$$

and

$$\left({}_{x_0}D_x^{\alpha}\right)[C]=0 \quad . \tag{4}$$

Definition 2.3 ([21])**:** If x, x_0, and a_n are real numbers for all n, $x_0\in(a,b)$, and $0<\alpha\leq 1$. If the function $f_\alpha:[a,b]\to R$ can be expressed as an α-fractional power series, that is, $f_\alpha(x^\alpha)=\sum_{n=0}^{\infty}\frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}$ on some open interval containing x_0, then we say that $f_\alpha(x^\alpha)$ is α-fractional analytic at x_0. Furthermore, if $f_\alpha:[a,b]\to R$ is continuous on closed interval $[a,b]$ and it is α-fractional analytic at every point in open interval (a,b), then f_α is called an α-fractional analytic function on $[a,b]$.

In the following, we introduce a new multiplication of fractional analytic functions.

Definition 2.4 ([22])**:** If $0<\alpha\leq 1$. Assume that $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ are two α-fractional power series at $x=x_0$,

$$f_\alpha(x^\alpha)=\sum_{n=0}^{\infty}\frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}, \tag{5}$$

$$g_\alpha(x^\alpha)=\sum_{n=0}^{\infty}\frac{b_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}. \tag{6}$$

Then

$$\begin{aligned}&f_\alpha(x^\alpha)\otimes_\alpha g_\alpha(x^\alpha)\\&=\sum_{n=0}^{\infty}\frac{a_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}\otimes_\alpha\sum_{n=0}^{\infty}\frac{b_n}{\Gamma(n\alpha+1)}(x-x_0)^{n\alpha}\\&=\sum_{n=0}^{\infty}\frac{1}{\Gamma(n\alpha+1)}\left(\sum_{m=0}^{n}\binom{n}{m}a_{n-m}b_m\right)(x-x_0)^{n\alpha}.\end{aligned} \tag{7}$$

Equivalently,

$$\begin{aligned}&f_\alpha(x^\alpha)\otimes_\alpha g_\alpha(x^\alpha)\\&=\sum_{n=0}^{\infty}\frac{a_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes_\alpha n}\otimes_\alpha\sum_{n=0}^{\infty}\frac{b_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes_\alpha n}\\&=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\sum_{m=0}^{n}\binom{n}{m}a_{n-m}b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes_\alpha n}.\end{aligned} \tag{8}$$

Definition 2.5 ([23])**:** If $0<\alpha\leq 1$, and x is a real number. The α-fractional exponential function is defined by

$$E_\alpha(x^\alpha)=\sum_{n=0}^{\infty}\frac{x^{n\alpha}}{\Gamma(n\alpha+1)}=\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha n}. \tag{9}$$

On the other hand, the α-fractional cosine and sine function are defined as follows:

$$cos_\alpha(x^\alpha)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n\alpha}}{\Gamma(2n\alpha+1)}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 2n}, \tag{10}$$

and

$$sin_\alpha(x^\alpha)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha (2n+1)}. \tag{11}$$

Definition 2.6: If $0<\alpha\leq 1$, and A is a matrix. The matrix α-fractional exponential function is defined by

$$E_\alpha(Ax^\alpha)=\sum_{n=0}^{\infty}A^n\frac{x^{n\alpha}}{\Gamma(n\alpha+1)}=\sum_{n=0}^{\infty}\frac{1}{n!}\left(A\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha n}. \tag{12}$$

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And the matrix α-fractional cosine and sine function are defined as follows:

$$cos_\alpha(Ax^\alpha) = \sum_{n=0}^{\infty} A^n \frac{(-1)^n x^{2n\alpha}}{\Gamma(2n\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(A\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 2n}, \tag{13}$$

and

$$sin_\alpha(Ax^\alpha) = \sum_{n=0}^{\infty} A^n \frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(A\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha (2n+1)}. \tag{14}$$

Theorem 2.7 (matrix fractional Euler's formula): *If* $0 < \alpha \le 1$, *and* A *is a real matrix, then*

$$E_\alpha(iAx^\alpha) = cos_\alpha(Ax^\alpha) + isin_\alpha(Ax^\alpha). \tag{15}$$

Theorem 2.8 (matrix fractional DeMoivre's formula)**:** *If* $0 < \alpha \le 1$, p *is an integer, and* A *is a real matrix, then*

$$[cos_\alpha(Ax^\alpha) + isin_\alpha(Ax^\alpha)]^{\otimes_\alpha p} = cos_\alpha(pAx^\alpha) + isin_\alpha(pAx^\alpha). \tag{16}$$

Theorem 2.9 (fractional binomial theorem)**:** *If* $0 < \alpha \le 1$, n *is a positive integer and* $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ *are two* α-*fractional analytic functions. Then*

$$[f_\alpha(x^\alpha) + g_\alpha(x^\alpha)]^{\otimes_\alpha \boldsymbol{n}} = \sum_{k=0}^{n}\binom{n}{k}\left(f_\alpha(x^\alpha)\right)^{\otimes_\alpha (n-k)} \otimes_\alpha \left(g_\alpha(x^\alpha)\right)^{\otimes_\alpha k}, \tag{17}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

III. MAIN RESULTS

In this section, we solve two matrix fractional integrals. At first, a lemma is needed.

Lemma 3.1: *If* $0 < \alpha \le 1$, r, s *are real numbers,* p *is a positive integer, and* A *is a real matrix, then*

$$E_\alpha(rAx^\alpha) \otimes_\alpha \left(cos_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p} = \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k} E_\alpha([r + i(p-2k)s]Ax^\alpha), \tag{18}$$

and

$$E_\alpha(rAx^\alpha) \otimes_\alpha \left(sin_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p} = \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k E_\alpha([r + i(p-2k)s]Ax^\alpha). \tag{19}$$

Proof Using matrix fractional Euler's f formula and matrix fractional DeMoivre's formula yields

$$E_\alpha(rAx^\alpha) \otimes_\alpha \left(cos_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p}$$

$$= E_\alpha(rAx^\alpha) \otimes_\alpha \left(\frac{1}{2}[E_\alpha(isAx^\alpha) + E_\alpha(-isAx^\alpha)]\right)^{\otimes_\alpha p}$$

$$= E_\alpha(rAx^\alpha) \otimes_\alpha \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}[E_\alpha(isAx^\alpha)]^{\otimes_\alpha (\boldsymbol{p}-\boldsymbol{k})} \otimes_\alpha [E_\alpha(-isAx^\alpha)]^{\otimes_\alpha \boldsymbol{k}}$$

$$= \frac{1}{2^p} E_\alpha(rAx^\alpha) \otimes_\alpha \sum_{k=0}^{p}\binom{p}{k} E_\alpha(i(p-k)sAx^\alpha) \otimes_\alpha E_\alpha(-iksAx^\alpha)$$

$$= \frac{1}{2^p} E_\alpha(rAx^\alpha) \otimes_\alpha \sum_{k=0}^{p}\binom{p}{k} E_\alpha(i(p-2k)sAx^\alpha)$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k} E_\alpha([r + i(p-2k)s]Ax^\alpha).$$

And

$$E_\alpha(rAx^\alpha) \otimes_\alpha \left(sin_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p}$$

$$= E_\alpha(rAx^\alpha) \otimes_\alpha \left(\frac{1}{2}[E_\alpha(isAx^\alpha) - E_\alpha(-isAx^\alpha)]\right)^{\otimes_\alpha p}$$

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$$= E_\alpha(rAx^\alpha)\otimes_\alpha \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}[E_\alpha(isAx^\alpha)]^{\otimes_\alpha (\boldsymbol{p-k})}\otimes_\alpha[-E_\alpha(-isAx^\alpha)]^{\otimes_\alpha \boldsymbol{k}}$$

$$= \frac{1}{2^p}E_\alpha(rAx^\alpha)\otimes_\alpha \sum_{k=0}^{p}\binom{p}{k}(-1)^k E_\alpha(i(p-k)sAx^\alpha)\otimes_\alpha E_\alpha(-iksAx^\alpha)$$

$$= \frac{1}{2^p}E_\alpha(rAx^\alpha)\otimes_\alpha \sum_{k=0}^{p}\binom{p}{k}(-1)^k E_\alpha(i(p-2k)sAx^\alpha)$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k E_\alpha([r+i(p-2k)s]Ax^\alpha)\,.$$

q.e.d.

Theorem 3.2: *If* $0<\alpha\le 1$, $(-1)^\alpha$ *exists,* r, s *are real numbers,* $r>0$, p *is a positive integer,* A *is a real matrix, and* A *is invertible, then*

$$\left({}_{-\infty}I_x^\alpha\right)\left[E_\alpha(rAx^\alpha)\otimes_\alpha \left(cos_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p}\right]$$

$$= \frac{1}{2^p}A^{-1}E_\alpha(rAx^\alpha)\otimes_\alpha \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^2+[(p-2k)s]^2}[rcos_\alpha((p-2k)sAx^\alpha)+[(p-2k)s]sin_\alpha((p-2k)sAx^\alpha)]\,, \quad (20)$$

and

$$\left({}_{-\infty}I_x^\alpha\right)\left[E_\alpha(rAx^\alpha)\otimes_\alpha \left(sin_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p}\right]$$

$$= \frac{1}{2^p}A^{-1}E_\alpha(rAx^\alpha)\otimes_\alpha \sum_{k=0}^{p}\binom{p}{k}(-1)^k\frac{1}{r^2+[(p-2k)s]^2}[rcos_\alpha((p-2k)sAx^\alpha)+[(p-2k)s]sin_\alpha((p-2k)sAx^\alpha)]. \quad (21)$$

Proof By Lemma 3.1,

$$\left({}_{-\infty}I_x^\alpha\right)\left[E_\alpha(rAx^\alpha)\otimes_\alpha \left(cos_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p}\right]$$

$$= \left({}_{-\infty}I_x^\alpha\right)\left[\frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}E_\alpha([r+i(p-2k)s]Ax^\alpha)\right]$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\left({}_{-\infty}I_x^\alpha\right)[E_\alpha([r+i(p-2k)s]Ax^\alpha)]$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\frac{1}{r+i(p-2k)s}A^{-1}[E_\alpha([r+i(p-2k)s]Ax^\alpha)]$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}\frac{r-i(p-2k)s}{r^2+[(p-2k)s]^2}A^{-1}\left[E_\alpha(rAx^\alpha)\otimes_\alpha[cos_\alpha((p-2k)sAx^\alpha)+isin_\alpha((p-2k)sAx^\alpha)]\right]$$

$$= \frac{1}{2^p}A^{-1}E_\alpha(rAx^\alpha)\otimes_\alpha \sum_{k=0}^{p}\binom{p}{k}\frac{1}{r^2+[(p-2k)s]^2}[rcos_\alpha((p-2k)sAx^\alpha)+[(p-2k)s]sin_\alpha((p-2k)sAx^\alpha)]\,.$$

And

$$\left({}_{-\infty}I_x^\alpha\right)\left[E_\alpha(rAx^\alpha)\otimes_\alpha \left(sin_\alpha(sAx^\alpha)\right)^{\otimes_\alpha p}\right]$$

$$= \left({}_{-\infty}I_x^\alpha\right)\left[\frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k E_\alpha([r+i(p-2k)s]Ax^\alpha)\right]$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\left({}_{-\infty}I_x^\alpha\right)[E_\alpha([r+i(p-2k)s]Ax^\alpha)]$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\frac{1}{r+i(p-2k)s}A^{-1}[E_\alpha([r+i(p-2k)s]Ax^\alpha)]$$

$$= \frac{1}{2^p}\sum_{k=0}^{p}\binom{p}{k}(-1)^k\frac{r-i(p-2k)s}{r^2+[(p-2k)s]^2}A^{-1}\left[E_\alpha(rAx^\alpha)\otimes_\alpha[cos_\alpha((p-2k)sAx^\alpha)+isin_\alpha((p-2k)sAx^\alpha)]\right]$$

$$= \frac{1}{2^p}A^{-1}E_\alpha(rAx^\alpha)\otimes_\alpha \sum_{k=0}^{p}\binom{p}{k}(-1)^k\frac{1}{r^2+[(p-2k)s]^2}[rcos_\alpha((p-2k)sAx^\alpha)+[(p-2k)s]sin_\alpha((p-2k)sAx^\alpha)]\,.$$

q.e.d.

IV. CONCLUSION

In this paper, based on Jumarie's modified R-L fractional integral and a new multiplication of fractional analytic functions, we obtain the exact solutions of two matrix fractional integrals. In addition, our results are generalizations of ordinary calculus results. In the future, we will continue to use Jumarie type of R-L fractional calculus and the new multiplication of fractional analytic functions to solve problems in applied mathematics and fractional differential equations.

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